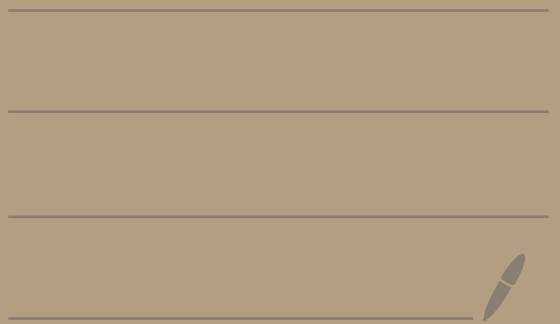


2550-01

Fall 2024

Test 1 Solutions



①

$$\underline{a=1, b=1}; \quad 1 \cdot \langle -1, 1, 2 \rangle + 1 \cdot \langle 2, 2, 0 \rangle = \langle 1, 3, 2 \rangle$$

$$\underline{a=0, b=1}; \quad 0 \cdot \langle -1, 1, 2 \rangle + 1 \cdot \langle 2, 2, 0 \rangle = \langle 2, 2, 0 \rangle$$

$$\underline{a=2, b=0}; \quad 2 \cdot \langle -1, 1, 2 \rangle + 0 \cdot \langle 2, 2, 0 \rangle = \langle -2, 2, 4 \rangle$$

②

$$(a) \quad 2\vec{a} - 3\vec{b} = 2\langle 1, -1 \rangle - 3\langle 2, 3 \rangle = \langle 2, -2 \rangle + \langle -6, -9 \rangle \\ = \langle -4, -11 \rangle$$

$$(b) \quad \|\vec{d}\| = \sqrt{1^2 + 2^2 + (-1)^2 + 2^2 + 0^2} = \sqrt{10}$$

$$(c) \quad \vec{a} \cdot \vec{b} = \langle 1, -1 \rangle \cdot \langle 2, 3 \rangle = 2 - 3 = -1$$

$$\vec{c} \cdot \vec{d} = \langle 2, 0, -1, 3, -1 \rangle \cdot \langle 1, 2, -1, 2, 0 \rangle$$

$$= 2 + 0 + 1 + 6 + 0$$

$$= 9$$

(3)

$$(a) -A + 2B = -\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} + 2\begin{pmatrix} -1 & 1 \\ 3 & -4 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -2 \\ -3 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 2 \\ 6 & -8 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 3 & -9 \end{pmatrix}$$

$$(b) BE = \begin{pmatrix} -1 & 1 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -1+3 & -2+4 \\ 3-12 & 6-16 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ -9 & -10 \end{pmatrix}$$

$$(c) CD = \begin{pmatrix} 1 & -1 & 0 \\ 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3-1+0 \\ 6-2+0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$(d) C^T = \begin{pmatrix} 1 & -1 & 0 \\ 2 & -2 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 \\ -1 & -2 \\ 0 & 1 \end{pmatrix}$$

④ See HW 3 # 4(a)

$$\begin{cases} x + z - w = 1 \\ y - z + w = 0 \\ w = 2 \end{cases}$$

Already reduced.
Leading: x, y, w
free: z

$$\begin{cases} x = 1 - z + w & \textcircled{1} \\ y = z - w & \textcircled{2} \\ w = 2 & \textcircled{3} \\ z = t & \textcircled{4} \end{cases}$$

① $z = t$

② $w = 2$

③ $y = z - w = t - 2$

④ $x = 1 - z + w = 1 - t + 2 = 3 - t$

Answer:

$$x = 3 - t$$

$$y = -2 + t$$

$$z = t$$

$$w = 2$$

⑥ See HW 2 - Part 2 # 1(f)