2550-01
Fall 2024
Test 1 Solutions

(1)

$a=1, b=1$: $1\cdot\langle -1,1,2\rangle + 1\cdot\langle 2,2,0\rangle = \langle 1,3,2\rangle$

$a=0, b=1$: $0\cdot\langle -1,1,2\rangle + 1\langle 2,2,0\rangle = \langle 2,2,0\rangle$

$a=2, b=0$: $2\cdot\langle -1,1,2\rangle + 0\langle 2,2,0\rangle = \langle -2,2,4\rangle$

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(2)

(a) $2\vec{a}-3\vec{b} = 2\langle 1,-1\rangle - 3\langle 2,3\rangle = \langle 2,-2\rangle + \langle -6,-9\rangle$

$= \langle -4,-11\rangle$

(b) $\|\vec{d}\| = \sqrt{1^2+2^2+(-1)^2+2^2+0^2} = \sqrt{10}$

(c) $\vec{a}\cdot\vec{b} = \langle 1,-1\rangle\cdot\langle 2,3\rangle = 2-3 = -1$

$\vec{c}\cdot\vec{d} = \langle 2,0,-1,3,-1\rangle\cdot\langle 1,2,-1,2,0\rangle$

$= 2+0+1+6+0$

$= 9$

(3)

(a) $-A+2B = -\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} + 2\begin{pmatrix} -1 & 1 \\ 3 & -4 \end{pmatrix}$

$= \begin{pmatrix} -1 & -2 \\ -3 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 2 \\ 6 & -8 \end{pmatrix} = \boxed{\begin{pmatrix} -3 & 0 \\ 3 & -9 \end{pmatrix}}$

(b) $BE = \begin{pmatrix} -1 & 1 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -1+3 & -2+4 \\ 3-12 & 6-16 \end{pmatrix} = \boxed{\begin{pmatrix} 2 & 2 \\ -9 & -10 \end{pmatrix}}$

(c) $CD = \begin{pmatrix} 1 & -1 & 0 \\ 2 & -2 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3-1+0 \\ 6-2+0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

(d) $C^T = \begin{pmatrix} 1 & -1 & 0 \\ 2 & -2 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 \\ -1 & -2 \\ 0 & 1 \end{pmatrix}$

④ See HW 3 #4(a)

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⑤

$$\begin{aligned} x \quad &+ z - w = 1 \\ y \; &- z + w = 0 \\ &\quad w = 2 \end{aligned}$$

Already reduced.
Leading: x, y, w
free: z

$$\begin{aligned} x &= 1 - z + w \quad (1) \\ y &= z - w \quad (2) \\ w &= 2 \quad (3) \\ z &= t \quad (4) \end{aligned}$$

① $z = t$

② $w = 2$

③ $y = z - w = t - 2$

④ $x = 1 - z + w = 1 - t + 2 = 3 - t$

Answer:

$$\begin{aligned} x &= 3 - t \\ y &= -2 + t \\ z &= t \\ w &= 2 \end{aligned}$$

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⑥ See HW 2 - Part 2 #1(f)